Consider the case of 2 risky assets

- Steps to form a combined portfolio using T-Bills, a risky stock fund (S), and a risky bond fund (B)
  1. Compute the characteristics of the assets
     - S: Expected return= 15%, standard deviation=15%
     - B: Expected return= 8%, standard deviation=8%
     - Correlation of the returns of S & B equals 0.2.
     - Risk-free rate equals 4%
  2. Trace out the efficient frontier of risky assets
  3. Identify the optimal risky portfolio by maximizing the Sharpe ratio of the capital allocation line (CAL)
  4. Identify the percentage invested in the optimal risky portfolio that maximizes the investor’s utility
Various combinations of the two risky assets when correlation = 0.2.

Find the capital allocation line with the maximum Sharpe ratio.
The optimal risky portfolio is the point where the CAL is tangent to the set of risky assets.

The investor will allocate his wealth between the optimal risky portfolio and T-Bills depending on his level of risk aversion.
Example 2: Two-Risky Assets

• Set-up:
  – Stock fund (S) and Bond Fund (B)
  – Two possible states of nature: good times and bad times – equal chance of each state
  – Holding Period Return (one period)
    • S: Good times=30%, bad times=-12%
    • B: Good times=-1%, bad times=15%

• Expected Return:
  – S: $E[r_S] = 0.5 \times 30\% + 0.5 \times (-12\%) = 9\%$
  – B: $E[r_B] = 0.5 \times (-1\%) + 0.5 \times (15\%) = 7\%$

Example: Two Risky Assets (cont.)

• Standard Deviation:
  – S: $\sigma_S^2 = 0.5 \times (30\%-9\%)^2 + 0.5 \times (-12\%-9\%)^2 = 4.41\%
    • $\sigma_S = 21\%$
  – B: $\sigma_B^2 = 0.5 \times (-1\%-7\%)^2 + 0.5 \times (15\%-7\%)^2 = 0.64\%
    • $\sigma_B = 8\%$

• Covariance
  – Cov($r_S, r_B$) = $0.5 \times (30\%-9\%) \times (-1\%-7\%) + 0.5 \times (-12\%-9\%) \times (15\%-7\%) = -1.68\%$

• Correlation
  – Corr($r_S, r_B$) = -1.68\% / (21\% \times 8\%) = -1
Example: Two Risky Assets (cont.)

• Portfolio Statistics:
  – Assume a weight of 60% in S and 40% in B
  – \( E[r_p] = 0.6 \times (9\%) + 0.4 \times (7\%) = 8.2\% \)
  – \( \sigma_p^2 = 0.6^2(4.41\%) + 0.4^2(0.64\%) + 2 \times 0.6 \times 0.4 \times (-1.68\%) \)
    = 0.8836\%
  – \( \sigma_p = 9.4\% \)

Example: Two Risky Assets (cont.)

• Investor Choice:
  – Investor will invest a portion of his assets \( y \) in the risky portfolio and a portion \( (1-y) \) in T-Bills
  – \( E[r_c] = yE[r_p] + (1-y)rf = rf + y(E[r_p] - rf) \)
  – \( \sigma_c^2 = y^2 \sigma_p^2 \)
  – Investor chooses the \( y \) that maximizes his utility
    • Suppose \( U = E[r_c] - \frac{1}{2}A\sigma_c^2 \)
      – \( A \) is a measure of the investor’s level of risk aversion
    • After substitution, \( U = rf + y(E[r_p] - rf) - \frac{1}{2}A y^2 \sigma_p^2 \)
    • At the max, \( y = (E[r_p] - rf) / (A\sigma_p^2) \)
Example: Two Risky Assets (cont.)

• The investor’s combined portfolio (rf = 5%):
  – If A=6, then $y = \frac{(8.2\% - 5\%)}{(6*0.8836\%)} = 60.36\%$
  – $E[r_c] = 5\% + (60.36\%)(8.2\%-5\%) = 6.93\%$
  – $\sigma_c = (60.36\%)(9.4\%) = 5.67\%$

• Comparison of measures:

<table>
<thead>
<tr>
<th></th>
<th>Expected Return</th>
<th>Standard Deviation</th>
<th>Sharpe Ratio</th>
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<tbody>
<tr>
<td>Stock Fund</td>
<td>9%</td>
<td>21%</td>
<td>.19</td>
</tr>
<tr>
<td>Bond Fund</td>
<td>7%</td>
<td>8%</td>
<td>.25</td>
</tr>
<tr>
<td>Risky Portfolio</td>
<td>8.2%</td>
<td>9.4%</td>
<td>.34</td>
</tr>
<tr>
<td>Combined Portfolio</td>
<td>6.93%</td>
<td>5.67%</td>
<td>.34</td>
</tr>
</tbody>
</table>

What Happens with n-Assets?

• Portfolio Optimization with n-Assets
  ➢ Class next week will involve 4 asset portfolio
  ➢ Much easier to develop excel skills and modelling

• Portfolio Performance Measures
  ➢ Excel Case defines metrics and methods to be used (assignment 1)
  ➢ Your job will be to build spreadsheet and analyze