Chapter 5

The Time Value of Money - The Basics

Chapter 5 Contents

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• Principles Used in this Chapter
  1. Using Time Lines
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  3. Discounting and Present Value
  4. Making Interest Rates Comparable
Learning Objectives

1. Construct cash flow timelines to organize your analysis of time value of money problems and learn three techniques for solving problems.

2. Understand compounding and calculate the future value of cash flow using math formulas, a financial calculator and an Excel worksheet.

3. Understand discounting and calculate the present value of cash flows using mathematical formulas, a financial calculator and an excel spreadsheet.

4. Understand how interest rates are quoted and how to make them comparable.

Foundation Principles Used in the Chapter

• Principle 1:
  - Money Has a Time Value.

• The concept of time value of money – a dollar received today, other things being the same, is worth more than a dollar received a year from now, underlies many financial decisions faced in business.

• This is a basic and fundamental tool to be used during the remainder of the course!!!
Using Timelines to Visualize Cashflows

- A **timeline** identifies the timing and amount of a stream of cash flows along with the interest rate.

- A timeline is typically expressed in years, but it could also be expressed as months, days or any other unit of time.

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Time Line Example

- The 4-year timeline illustrates the following:
  - The interest rate is 10%.
  - A cash outflow of $100 occurs at the beginning of the first year (at time 0), followed by cash inflows of $30 and $20 in years 1 and 2, a cash outflow of $10 in year 3 and cash inflow of $50 in year 4.
Checkpoint 5.1

Creating a Timeline
Suppose you lend a friend $10,000 today to help him finance a new Jimmy John’s Sub Shop franchise and in return he promises to give you $12,155 at the end of the fourth year. How can one represent this as a timeline? Note that the interest rate is 5%.

---

**STEP 1: Picture the problem**
A timeline provides a tool for visualizing cash flows and time:

- **Time Period:** 0 1 2 3 4 **Years**
- **Cash Flow:** Year 0 Year 1 Year 2 Year 3 Year 4

**STEP 2: Decide on a solution strategy**
To complete the timeline, we simply record the cash flows onto the template.

**STEP 3: Solve**
We can input the cash flows for this investment on the timeline as shown below. Time period zero (the present) is shown at the left end of the timeline, and future time periods are shown above the timeline, moving from left to right, with the year that each cash flow occurs shown above the timeline.
Keep in mind that year 1 represents the end of the first year as well as the beginning of the second year.

- **Time Period:** 0 1 2 3 4 **Years**
- **Cash Flow:** $-10,000 $12,155

**STEP 4: Analyze**
Using timelines to visualize cash flows is useful in financial problem solving. From analyzing the timeline, we can see that there are two cash flows, an initial $10,000 cash outflow, and a $12,155 cash inflow at the end of year 4.
Checkpoint 5.2: Check yourself

Draw a timeline for an investment of:
• $40,000 today that returns
• nothing in one year,
• $20,000 at the end of year 2,
• nothing in year 3,
• and $40,000 at the end of year 4.

Timeline

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$40,000</td>
</tr>
<tr>
<td>1</td>
<td>$0</td>
</tr>
<tr>
<td>2</td>
<td>$20,000</td>
</tr>
<tr>
<td>3</td>
<td>$0</td>
</tr>
<tr>
<td>4</td>
<td>$40,000</td>
</tr>
</tbody>
</table>

i=interest rate; not given
Simple Interest and Compound Interest

- **Simple interest**: Interest is earned only on the principal amount.

- **Compound interest**: Interest is earned on both the principal and accumulated interest of prior periods.

**Example 5.1**: Suppose that you deposited $500 in your savings account that earns 5% annual interest. How much will you have in your account after two years using:
  - (a) simple interest and
  - (b) compound interest?

**Example 5.1 (cont.)**

- **Simple Interest**
  - Interest earned
    - $5% of $500 = .05 \times 500 = $25 per year
  - Total interest earned = $25 \times 2 = $50
  - Balance in your savings account:
    - $500 + $50 = $550

- **Compound interest**
  - Interest earned in Year 1 = 5% of $500 = $25
  - Interest earned in Year 2
    - $5% of ($500 + accumulated interest)
    - $5% of ($500 + 25) = .05 \times 525 = $26.25
  - Balance in savings account = Principal + interest earned
    - $500 + $25 + $26.25 = $551.25
Present Value and Future Value

- Time value of money calculations involve
  - **Present value** (what a cash flow would be worth to you today) and
  - **Future value** (what a cash flow will be worth in the future).

- In example 5.1,
  - Present value is $500 and
  - Future value is $551.25

Future Value Equation 5-1a

We can apply equation 5-1a to example 5.1

\[
FV_2 = PV(1+i)^n
\]

\[
= 500(1.05)^2
\]

\[
= \$551.25
\]

- Continue example 5.1 where you deposited $500 in savings account earning 5% annual interest.
- Show the amount of **interest earned for the first five years** and the **value of your savings at the end of five years**.
Future Value Example (cont.)

<table>
<thead>
<tr>
<th>YEAR</th>
<th>PV or Beginning Value</th>
<th>Interest Earned (5%)</th>
<th>FV or Ending Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$500.00</td>
<td>$25</td>
<td>$525</td>
</tr>
<tr>
<td>2</td>
<td>$525.00</td>
<td>$26.25</td>
<td>$551.25</td>
</tr>
<tr>
<td>3</td>
<td>$551.25</td>
<td>$27.56</td>
<td>$578.81</td>
</tr>
<tr>
<td>4</td>
<td>$578.81</td>
<td>$28.94</td>
<td>$607.75</td>
</tr>
<tr>
<td>5</td>
<td>$607.75</td>
<td>$30.39</td>
<td>$638.14</td>
</tr>
</tbody>
</table>

We will obtain the same answer using equation 5-1

\[
FV = PV (1+i)^n
\]

\[
= 500(1.05)^5
\]

\[= $638.14\]

Power of Time

Figure 5.1 Future Value and Compound Interest Illustrated

[Panel II] The Power of Time

This figure illustrates the importance of time when it comes to compounding. Because interest is earned on past interest, the future value of $100 deposited in an account that earns 6% compounded annually grows over time. For example, if you were to expand this figure to 45 years (which is about how long you have until you retire, assuming you're around 20 years old right now), it would grow to over 31 times its initial value.
Main Take-Aways From Panel B & C of Fig. 5-1

1. **Power of Time**: Future value of original investment increases with time. However, there will be no change in future value if the interest rate is equal to zero.

2. **Power of Rate of Interest**: An increase in rate of interest leads to an increase in future value.
Checkpoint 5.2: Future Value of a Cash Flow

You are put in charge of managing your firm’s working capital. Your firm has $100,000 in extra cash on hand and decides to put it in a savings account paying 7% interest compounded annually.

- How much will be in the account in 10 years?

**STEP 1: Picture the problem**

We can set up a timeline to identify the cash flows from the investment as follows:

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Cash Flow</th>
<th>Future Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-100,000</td>
<td>FV_{10}</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
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<tr>
<td>4</td>
<td></td>
<td></td>
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<tr>
<td>5</td>
<td></td>
<td></td>
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<tr>
<td>6</td>
<td></td>
<td></td>
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<tr>
<td>7</td>
<td></td>
<td></td>
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<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>FV_{10}</td>
</tr>
</tbody>
</table>

**STEP 2: Decide on a solution strategy**

This is a simple future value problem. We can find the future value using Equation 5-1a.

\[
FV_{n} = P \left(1 + \frac{i}{100}\right)^n
\]

\[
P = 100,000, \quad i = 7\%, \quad n = 10 \text{ years}
\]

\[
FV_{10} = 100,000 \left(1 + \frac{0.07}{100}\right)^{10}
\]

\[
FV_{10} = 100,000 \times (1.0715)
\]

\[
FV_{10} = 196,715
\]

At the end of ten years, you will have $196,715 in the savings account.

**Checkpoint 5.2 – First Financial Calculator Lesson**

**STEP 3: Solve**

Using the Mathematical Formulas. Substituting \( P = 100,000, \ i = 7\% \), and \( n = 10 \) years into Equation (5-1a), we get

\[
FV_{n} = P \left(1 + \frac{i}{100}\right)^n
\]

\[
P = 100,000, \quad i = 7\%, \quad n = 10 \text{ years}
\]

\[
FV_{10} = 100,000 \left(1 + \frac{0.07}{100}\right)^{10}
\]

\[
FV_{10} = 100,000 \times (1.0715)
\]

\[
FV_{10} = 196,715
\]

At the end of ten years, you will have $196,715 in the savings account.

Using a Financial Calculator.

Enter

<table>
<thead>
<tr>
<th>10</th>
<th>7.0</th>
<th>-100,000</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>INT</td>
<td>PV</td>
<td>PMT</td>
</tr>
</tbody>
</table>

Solve for

196,715

Using an Excel Spreadsheet.

\( = FV(rate,\text{nper},\text{pmt},0) \) or with values entered \( = FV(0.07,10,0,-100000) \)
FV Applications in Other Areas

- **Example 5.2** A DVD rental firm is currently renting 8,000 DVDs per year.
  - How many DVDs will the firm be renting in 10 years if the demand for DVD rentals is expected to increase by 7% per year?

  • Using Equation 5-1a,
    \[ FV = 8000(1.07)^{10} = \$15,737.21 \]

  • Using Financial Calculator
    \[ N = 10 \quad I/Y = 7 \quad PV = -8,000 \quad PMT = 0 \]
    Compute \( FV = ? \)

Compound Interest with Shorter Compounding Periods

- Banks frequently offer savings account that compound interest every day, month, or quarter.
- More frequent compounding will generate higher interest income for savers if the APR is the same.
- **Example 5.4** You invest $500 for seven years to earn an annual interest rate of 8%, and the investment is compounded semi-annually. What will be the future value of this investment?
- Use equation 5-1b to solve the problem.
  - Adjusts the number of compounding periods and interest rate to reflect the semi-annual compounding.
Compound Interest with Shorter Compounding Periods (cont.)

\[
FV = PV \left(1 + \frac{i}{2}\right)^{m*2}
\]

\[
= 500 \left(1 + \frac{.08}{2}\right)^{7*2}
\]

\[
= 500(1.7317)
\]

\[
= $865.85
\]

Financial Calculator: I/Y = 4%, N=14
Checkpoint 5.3

Calculating Future Values Using Non-Annual Compounding Periods

You have been put in charge of managing your firm’s cash position and noticed that the Plaza National Bank of Portland, Oregon, has recently decided to begin paying interest compounded semi-annually instead of annually. If you deposit $1,000 with Plaza National Bank at an interest rate of 12%, what will your account balance be in five years?

**STEP 1: Picture the problem**

If you earn a 12% annual rate compounded semi-annually for five years, you really earn 6% every six months for 10 six-month periods. Expressed as a timeline, this problem would look like the following:

\[
\begin{array}{c|ccccccccccc}
\text{Time Period} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
\text{Cash Flow} & -1,000 \\
\end{array}
\]

**STEP 2: Decide on a solution strategy**

In this instance we are simply solving for the future value of $1,000. The only twist is that interest is calculated on a semi-annual basis. Thus, if you earn 12% compounded semi-annually for five years, you really earn 6% every six months for 10 six-month periods. We can calculate the future value of the $1,000 investment using Equation (5-1b).
Checkpoint 5.3

**Step 3: Solve**

Using the mathematical formulas, substituting number of years \( n = 5 \)
number of compounding periods per year \( m = 2 \)
annual interest rate \( i = 12\% \)
and \( PV = \$1,000 \) into Equation (5–19).

\[
\text{Future Value in year } n (FV_n) = \text{Present Value} \cdot \left(1 + \frac{\text{Annual Interest Rate}}{\text{Compounding Periods per Year}}\right)^{nm}
\]

Future Value in year 5 \( (FV_5) = \$1,000 \cdot \left(1 + \frac{12\%}{2}\right)^{10} = \$1,000 \times 1.79085 = \$1,790.85
\]

Using a Financial Calculator:

Enter: \( N \) \( 60 \) \( PMT \) \( 0 \)

Solve for: \( FV \)

You will have \$1,790.85 at the end of five years.

Using an Excel Spreadsheet:

= \( FV(i\%per,\text{per},\text{PV},\text{PMT},\text{type}) \)

or with values entered = \( FV(0.0012,10,0,-1000) \)

**Step 4: Analyze**

The more often interest is compounded per year—that is, the larger \( m \) is, resulting in a larger value of \( n \)—the larger the future value will be. That’s because you are earning interest more often on the interest you’ve previously earned.

Checkpoint 5.3: Another Example

If you deposit $50,000 in an account that pays an annual interest rate of 10% compounded monthly, what will your account balance be in 10 years?

**Step 1:** Picture the Problem (make a timeline)

**Step 2:** Decide on a Solution Strategy (Problem Recognition)

- This involves solving for future value of $50,000
Step 3: Solve

- **Using Mathematical Formula**

  \[
  FV = PV \left(1 + \frac{i}{12}\right)^{12m}
  \]

  \[
  = \$50,000 \left(1 + \frac{0.10}{12}\right)^{10 \times 12}
  \]

  \[
  = \$50,000 \times (2.7070)
  \]

  \[
  = \$135,352.07
  \]

- **Using a Financial Calculator**

  **Enter:** N = 120; I/Y = .833%; PMT = 0

  PV = -50,000 Solve FV = **$135,298.39**

Step 3: Solve (cont.) and Analyze

- **Using an Excel Spreadsheet**

  \[
  =FV(rate,nper,pmt, pv)
  \]

  \[
  =FV(0.00833,120, 0,-50000) = \$135,346.71
  \]

Step 4: Analyze

- More frequent compounding leads to a higher FV as you are earning interest more often on interest you have previously earned.

- If the interest was compounded annually, the FV would have been equal to:

  \[
  - \$50,000 \times (1.10)^{10} = \$129,687.12
  \]
Present Values: The Key Question

- What is value today of cash flow to be received in the future?

  - The answer to this question requires computing the present value i.e. the value today of a future cash flow, and the process of discounting, determining the present value of an expected future cash flow.

PV Equation

The term in the bracket is known as the Present Value Interest Factor (PVIF).

- PV = FVn PVIF
- Example 5.5 How much will $5,000 to be received in 10 years be worth today if interest rate is 7%?

  - PV = FV (1/(1+i)n) = 5000 (1/(1.07)10)
    = 5000 (.5083)
    = $2,541.50
Impact of Interest Rates on PV

- If the interest rate (or discount rate) is higher (say 9%), the PV will be lower.

\[
PV = 5000 \times \frac{1}{(1.09)^{10}} = 5000 \times (0.4224) = \$2,112.00
\]

- If the interest rate (or discount rate) is lower (say 2%), the PV will be higher.

\[
PV = 5000 \times \frac{1}{(1.02)^{10}} = 5000 \times (0.8203) = \$4,101.50
\]
Checkpoint 5.4 – Solve for Present Value

Your firm has just sold a piece of property for $500,000, but under the sales agreement, it won’t receive the $500,000 until ten years from today. What is the present value of $500,000 to be received ten years from today if the discount rate is 6% annually?

**Step 1: Picture the problem**
Expressed as a timeline, this problem would look like the following:

- **Time Period**: 0 1 2 3 4 5 6 7 8 9 10 **Years**
- **Cash Flow**
  - Present Value = ?
  - $500,000

**Step 2: Decide on a solution strategy**
In this instance, we are simply solving for the present value of $500,000 to be received at the end of 10 years. We can calculate the present value of the $500,000 using Equation (5-2).

**Step 3: Solve**

Using the Mathematical Formula: Substituting $PV_0 = $500,000, n = 10, and i = 6% into Equation (5-2), we find

$$PV = \frac{500,000 \times \frac{1}{1 + 0.06^{10}}}{1}$$

$$= \frac{500,000 \times 0.558394}{1}$$

$$= \frac{279,197}{1}$$

The present value of the $500,000 to be received in ten years is $279,197. Earlier we noted that discounting is the reverse of compounding. We can easily test this calculation by considering this problem in reverse. What is the future value in ten years of $279,197 today if the rate of interest is 6%? Using our PV Equation (5-1), we can see that the answer is $500,000.

Using a Financial Calculator:

<table>
<thead>
<tr>
<th>Enter</th>
<th>10</th>
<th>6.0</th>
<th>0</th>
<th>500,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve for</td>
<td>(PMT)</td>
<td>(FV)</td>
<td>(i)</td>
<td>(N)</td>
</tr>
</tbody>
</table>

Using an Excel Spreadsheet:

- \(PV(\text{rate}, \text{per}, \text{pmt}, \text{fv})\) or with values entered: \(PV(0.06, 10, 0, 500000)\)
Checkpoint 5.4: Class Work

What is the present value of $100,000 to be received at the end of 25 years given a 5% discount rate?

Solving for the Number of Periods

Key Question: How long will it take to accumulate a specific amount in the future?

- Easier to solve for “n” using financial calculator or Excel rather than mathematical formula.
- Example 5.6 How many years will it take for an investment of $7,500 to grow to $23,000 if it is invested at 8% annually?
  - Using a Financial Calculator

Enter:

\[ I/Y = 8 \quad PMT = 0 \quad PV = -7,500 \quad FV = 23,000 \]

Solve \( N = 14.56 \) periods (years)
Checkpoint 5.5 – In class Problem

Solving for the Number of Periods, \( n \)
Let’s assume that the Toyota Corporation has guaranteed that the price of a new Prius will always be $20,000, and you’d like to buy one but currently have only $7,752. How many years will it take for your initial investment of $7,752 to grow to $20,000 if it is invested so that it earns 9% compounded annually?

Checkpoint 5.5: Class Problem

How many years will it take for $10,000 to grow to $200,000 given a 15% compound growth rate?
Rule of 72 – Interesting Trivia/shortcut

- Rule of 72 is an approximate formula to determine the number of years it will take to double the value of your investment.

- Rule of 72: \( N = \frac{72}{\text{interest rate}} \)
- Example 5.7 Using Rule of 72, determine how long it will take to double your investment of $10,000 if you are able to generate an annual return of 9%.

\[
N = \frac{72}{\text{interest rate}} = \frac{72}{9} = 8 \text{ years}
\]

Solving for Rate of Interest

Key Question: What rate of interest will allow your investment to grow to a desired future value?

- Can determine the rate of interest using mathematical equation (a real pain), the financial calculator or the Excel spread sheet.

- Example 5.8 At what rate of interest must your savings of $10,000 be compounded annually for it to grow to $22,000 in 8 years (what return do you need to earn)?
Using Mathematical Formula:

\[ I = (FV/PV)^{1/n} - 1 \]
\[ = (22000/10000)^{1/8} - 1 \]
\[ = (2.2)^{.125} - 1 \]
\[ = 1.1035 - 1 \]
\[ = .1035 \text{ or } 10.35\% \]

Using Financial Calculator

Enter:

\[ N = 8; \quad PMT = 0; \quad PV = -10,000 \]
\[ FV = 22,000 \]
\[ \text{Solve } I/Y = 10.36 \text{ percent} \]

Checkpoint 5.6 – In Class Problem

Solving for the Interest Rate, \( i \)

Let’s go back to that Prius example in Checkpoint 5.5. Recall that the Prius always costs $20,000. In 10 years, you’d really like to have $20,000 to buy a new Prius, but you only have $11,167 now. At what rate must your $11,167 be compounded annually for it to grow to $20,000 in 10 years?
Checkpoint 5.6: Classroom Problem

At what rate will $50,000 have to grow to reach $1,000,000 in 30 years?

Annual Percentage Rate (APR)

- The **annual percentage rate** (APR) indicates the amount of interest paid or earned in one year without compounding. APR is also known as the nominal or stated interest rate. This is the rate required by law.
Comparing Loans using EAR

- Cannot compare two loans based on APR if they do not have the same compounding period.
- To make them comparable, calculate their equivalent rate using an annual compounding period. We do this by calculating the **effective annual rate (EAR)**

\[
\text{Effective Annual Rate (EAR)} = \left(1 + \frac{\text{Quoted Annual Rate}}{\text{Compounding Periods per year (m)}}\right)^m - 1
\]

Comparing Loans using EAR - Example

- **Example 5.9** Calculate the EAR for a loan that has a 5.45% quoted annual interest rate compounded monthly.

\[
\text{EAR} = [1 + .0545/12]^12 - 1
\]

\[
= 1.0558 - 1
\]

\[
= .05588 \text{ or } 5.59\%
\]

- Alternative Method (Financial Calculator)
  
  \[
  \text{PV} = -100; \ N = 12; \ I/Y = 5.45/12; \ PMT = 0
  \]
  
  Solve \( FV = 105.59 \) (Logic leads to 5.59% EAR)
Checkpoint 5.7 – Classroom Problem

Calculate an EAR or Effective Annual Rate

Assume that you just received your first credit card statement and the APR, or annual percentage rate listed on the statement, is 21.7%. When you look closer you notice that the interest is compounded daily. What is the EAR, or effective annual rate, on your credit card?

Continuous Compounding

• When the time intervals between when interest is paid are infinitely small, we can use the following mathematical formula to compute the EAR.

\[ \text{EAR} = (e^{\text{quoted rate}}) - 1 \]

Where “e” is the number 2.71828

• Continuous Compounding comes from really "mathy" finance theory but is easiest to use.

• Example: What is the EAR on credit card with continuous compounding if the APR is 18%?

\[ \text{EAR} = e^{0.18} - 1 = 1.1972 - 1 = 0.1972 \text{ or } 19.72\% \]